

Model exam topics Logic and Discrete Structures

January 2023

Part I

1. What is the contrapositive of the sentence "If it rains, it means I'll take my umbrella"?
 - a. "If it doesn't rain it implies that I don't take my umbrella."
 - b. "If I don't take my umbrella it implies that it doesn't rain."
 - c. "If it rains it means I don't take my umbrella."
 - d. "If I take my umbrella it implies that it is raining."

2. Let the function $f : A \rightarrow B$ and f is injective. Select all variants that are true:
 - a. $|A| \leq |B|$
 - b. $|A| \geq |B|$
 - c. $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
 - d. $x_1, x_2 \in A, x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

3. Select from the choices below which are the advantages of recursion in programming:
 - a. Complex problems can be divided into simpler subproblems and thus easier to solve
 - b. Repeated recursive calls use little memory
 - c. Repeated recursive calls use a lot of memory
 - d. Code is short and easy to follow, elegant, clean

4. Let A and B be two finite sets such that $|A|=3$ and $|B|=5$. How many functions $f : A \rightarrow B$ exist?

5. Let the relation $R = \{ (1, 2), (2, 1), (3, 3), (2, 3), (3, 2) \}$.
Which of the statements below is NOT true?
Select one:
 - a. The relationship is not transitive.
 - b. The relationship is not an ordering relationship.
 - c. The relationship is symmetrical.
 - d. The relationship is reflexive.

6. Select from the list below all the characteristics of a strict order relationship and only these.
 - a. transitive
 - b. irreflexive
 - c. symmetric
 - d. reflexive
 - e. antisymmetric

7. Which of the following statements about propositional logic is NOT true?
 - a. A formula in propositional logic has a finite number of interpretations.

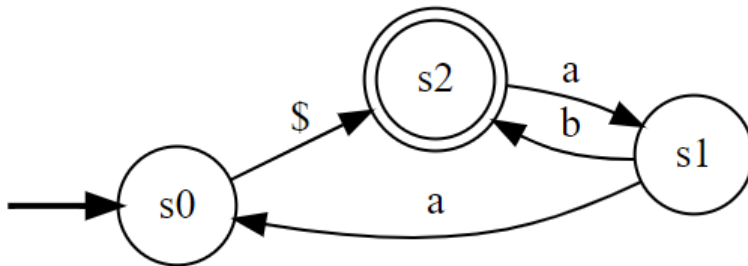
- b. Propositional logic is consistent.
- c. In propositional logic not every tautology is a theorem.

8. Select all the true statements below, and only these.

- a. In predicate logic any conclusion deduced from a set of hypotheses is true in any interpretation that satisfies all the hypotheses.
- b. In predicate logic any true statement can be proved.
- c. In predicate logic we can write statements that can neither be proved nor disproved.

9. Give the automaton in the figure below. Which of the regular expressions below recognizes the same language?

Note: denote the empty string with \$, and the alternation with +



- a. $\$(a^*+b^*)a b^*$
- b. $\$+a+(ab^*)a^*$
- c. $\$(a+b)^*a$
- d. $\$+a((b+a)a)^*(b+a)$

10. For the given grammar, which of the statements below is true?

Note: the empty string is denoted by \$, and the terminal set is { a, b }

$S \rightarrow a S b a S$

$S \rightarrow a b S$

$S \rightarrow b$

- a. The defined language is finite.
- b. The defined language contains the string aabbbab.
- c. The defined language does not contain the string abbaabb.
- d. The defined language does not contain the string ababb.

11. Which of the following statements is NOT true?

- a. A Turing Machine stops at the end of the input string.
- b. There is no known effective computation method more powerful than the Turing Machine
- c. A Turing Machine has unlimited memory.
- d. A Turing Machine can perform transformations on an input string.

12. Which answer generates the following sequence in Python?

```
a = (1, 2, 3)
```

```
b = (4, 5, 6)
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```
print(a.update(b))
```

- a. Error
- b. (1, 2, 3, 4, 5, 6)
- c. (1, 2, 3, (4, 5, 6))
- d. (4, 5, 6, 1, 2, 3)
- e. ((4, 5, 6), 1, 2, 3)

13. Select all the false statements below, the statements are about lists in Python:

- a. Lists are finite, but can have any length
- b. the order of elements does not matter
- c. a list can have no elements
- d. lists cannot be defined recursively

14. What is the identity property of the operations of a boolean algebra of sets?

- a. $A \cap A^c = \emptyset$
- b. $A \cup \emptyset = A$
- c. $A \cap A = A$
- d. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

15. Translate the following formula into normal-conjunctive form (CNF):

$$\neg((c \rightarrow \neg b) \wedge (a \rightarrow \neg c))$$

16. Formalize the following natural language sentences into formulas in Predicate Logic:

- a. There are professors in Computer Science who are demanding and will give all the hard subjects on exams.
- b. Children who have not been good will not get toys.

17. Construct a deterministic or non-deterministic finite automaton (your choice) that accepts all strings of a, b and c that start with ab and do not end with b. Write the related regular expression.

Part II - Problems to solve in the PYTHON programming language

1. Write a recursive function in Python that takes a natural number as a parameter and returns a string representing the binary conversion of the number given as a parameter.

Example: For n=5 return "101".

Procedure: To convert from decimal to binary, first divide the chosen number by 2; the remainder represents the least significant (rightmost) digit of the conversion result. The quotient is divided by 2, the remainder is noted, and the procedure is repeated with the new quotient. The operation ends when the quotient becomes zero.

2. Write a function that takes 2 lists as parameters. It turns the two lists into two sets and then returns a set that is the intersection of the 2 sets.

Example:

Input: [1, 7, 8, 10], [3, 4, 7, 8]

Output: {7, 8}

3. Write a function that takes parameter two lists (the first list has all distinct elements) and returns a dictionary that has keys from the first list and values from the second list. If the lists are of different lengths, the dictionary will have a number of elements equal to the number of elements in the shorter list.

Example:

Input: [1, 18, 118], [0, 1, 1, 1]

Output: {1: 0, 18: 1, 118: 1}